Ex5a

Exercise 9

9.1 The AC-Stark Shift

As shown in the lecture, the atom-light interaction for a two-level system can be described in the rotating light frame by

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\tilde{c}_1(t)\\ \tilde{c}_2(t)\end{pmatrix} = \frac{i}{2}\begin{pmatrix}-\delta & \Omega_0\\ \Omega_0 & \delta\end{pmatrix}\begin{pmatrix}\tilde{c}_1(t)\\ \tilde{c}_2(t)\end{pmatrix},$$

leading to the occupation probabilities $|\tilde{c}_1(t)|^2$, $|\tilde{c}_2(t)|^2$ for the ground and excited state (with Rabi frequency Ω_0 and detuning δ). In addition to its effect on the populations of the states, the light field also induces a change of energy levels usually referred to as the AC-Stark shift or light shift.

Naming: The corresponding eigenstates $|-\rangle$ and $|+\rangle$ for the coupled atom-light system are called 'dressed' states – in contrast to the 'bare' states $|1\rangle$ and $|2\rangle$.

(a) From the equation given above, determine the two eigenvalues of the coupled atom-light system E_- and E_+.

(b) Sketch the energies E_-, E_+ as a function of the detuning δ and compare them to E_1, E_2. Discuss the cases $\delta < 0$ and $\delta > 0$.

(c) Assume that the intensity of the light field varies with z, i.e. $I = I(z)$. Show that the atom in ground state experiences a force proportional to the intensity gradient of the light field in the far off-resonant regime $\Omega_0 \ll |\delta|$.

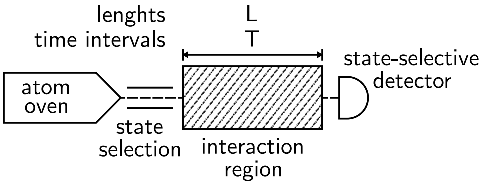
9.2 Cesium Atomic-Beam Clocks

The microwave transition frequency $6\mathrm{S}_{1/2}(F = 3, m_F = 0) \leftrightarrow 6\mathrm{S}_{1/2}(F = 4, m_F = 0)$ in atomic cesium-133 at approx. 9.2 GHz acts as the primary standard for the definition the second in the International System of Units (SI).

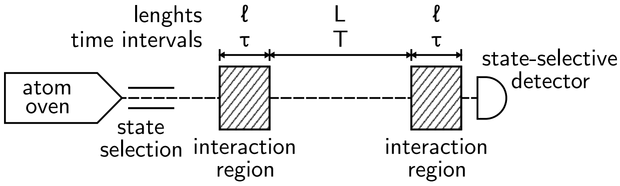
In the following we want to take a look at two simplified experimental setups for building an atomic cesium clock based on an hot atomic beam: The Rabi setup (i) features a single microwave interaction region of length L, whereas the Ramsey setup (ii) uses two

$\pi/2$-pulses with a free-flight region of length L. Actual realizations of the latter method are for the example the atomic clocks CS1 and CS2 at the PTB[1].

(i) Rabi setup



(ii) Ramsey setup



The atoms are initially preprared in ground state and unless stated otherwise, we assume a monochromatic beam of atoms (i.e. all atoms have the same velocity) and neglect spontaneous emission. For both the Rabi setup (i) and the Ramsey setup (ii):

(a) Visualize the experiments using the Bloch vector. How can the transition frequency be identified? Distinguish between an on-resonance microwave field ($\delta = 0$) and a (near) off-resonant excitation with $\delta \neq 0$.

(b) Using the model setups, determine the occupation probability $\tilde{\rho}_{22}$ of the excited state probed by the state-selective detector.

(i) For the Rabi setup, use the optical Bloch equations in Bloch vector form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} u(t) \\ v(t) \\ w(t) \end{pmatrix} = -\begin{pmatrix} \Omega_0 \\ 0 \\ \delta \end{pmatrix} \times \begin{pmatrix} u(t) \\ v(t) \\ w(t) \end{pmatrix}.$$

(ii) For the Ramsey setup, set up the system of differential equations describing the ground and excited state's coefficients $\tilde{c}_1(t)$ and $\tilde{c}_2(t)$ throughout the apparatus. You don't need to solve it[2], the final occupation propability at the end of the second interaction region reads

$$\tilde{\rho}_{22} = 4\frac{\Omega_0^2}{\Omega^2}\sin^2\left(\frac{\Omega}{2}\tau\right)\left[\Omega\cos\left(\frac{\delta}{2}T\right)\cos\left(\frac{\Omega}{2}\tau\right) - \delta\sin\left(\frac{\delta}{2}T\right)\sin\left(\frac{\Omega}{2}\tau\right)\right]^2$$

with generalized Rabi frequency $\Omega = \sqrt{\Omega_0^2 + \delta^2}$.

(c) Using the expressions for the occupation probabilities of the excited state $\tilde{\rho}_{22}$, justify your considerations in part (a). Plot the probabilities in dependence of the detuning δ assuming $\Omega_0 = 2\pi \times 125\,\mathrm{Hz}$ and realistic parameters for the apparatus.

[1] PTB = "Physikalisch-Technische Bundesanstalt" – the German national metrology institute.

[2] It's not complicated, just a bit lenghty...

How does the lineshape change for shorter/longer times T and what does this imply for the experimental feasibility of the two methods?

(d) Explain the effect of atoms with different velocities and write down an ansatz for the lineshape expected due to an atomic ensemble with a general velocity distribution $f(v)$.

We wish you Happy Holidays
and all the best for the New Year!